

Method for Approximating Magnetic Core Power-Loss Density

by Dennis L Feucht

Magnetic cores used in switching power converters must be sized in the design of transducers (magnetic devices). How this is done is often empirical and based on previous experience. Magnetics design programs and inexperienced engineers cannot resort to this method and must have a more analytic basis for determining core size, as measured in core volume. Manufacturers give rules of thumb of 5 W to 15 W of conversion power per cm^3 for most core shapes (and 30 W/ cm^3 for toroids). This can be used in a computer program for choosing a core, though it is somewhat crude.

Core specifications include a chart or algebraic function for power lost in the core as the power-loss density, p_c . It depends on peak magnetic-field density, \hat{B} , (that is, the amplitude of B) and frequency. The range of p_c on these charts covers several decades. Usually the switching frequency is determined by circuit design considerations, leaving a choice for \hat{B} . This choice immediately determines p_c . What value should be chosen?

The “Ohm’s Law” of thermal analysis is:

$$R_\theta \cdot P_c = \Delta T$$

If ΔT depends on core temperature limits and the temperature range over which the circuit is specified to operate, then ΔT is given. The total power dissipated in the core follows readily as:

$$P_c = p_c \cdot V$$

where, V is core volume.

The thermal resistance can be modeled as two resistances in series: the core resistance, R_c and the core-to-ambient resistance, R_{cA} . Then:

$$R_\theta = R_c + R_{cA}$$

Given a maximum ΔT and core power loss, P_c , then maximum thermal resistance is:

$$R_\theta = \frac{\Delta T}{P_c}$$

The thermal resistance of the core can be expressed using the basic formula for thermal resistance for conductive heat transfer, which is of the same form as that for electrical resistance:

$$R_\theta = \frac{l}{\sigma_\theta \cdot A}$$

where, σ_θ = thermal conductivity, l is the thermal path length, and A is the thermal path cross-sectional area.

This general expression for R_θ can be made more specific by choosing some geometry for the core which determines l and A . To minimize R_θ , l should be minimized relative to A . To make a general approximation, a simple shape somewhat like real cores should be chosen. This excludes complicated, heat-sink shapes with

many protruding fins. The optimal simple thermal core shape, which maximizes surface area to volume, is the sphere, for which:

$$A = 4 \cdot \pi \cdot r^2 ; V = \frac{4}{3} \cdot \pi \cdot r^3$$

where, r is the radius.

For a sphere, the characteristic length of the thermal path is taken to be $l = r$. Then the thermal resistance of the core is approximated ideally as:

$$R_c \cong \frac{r}{\sigma_c \cdot (4 \cdot \pi \cdot r^2)} = \frac{1}{\sigma_c \cdot 4 \cdot \pi \cdot r}$$

From core manufacturer data, R_c decreases with increasing V . This agrees with the expression for R_c . The thermal model is that of a sphere producing heat uniformly throughout its interior and convection heat from it at its surface to the surrounding air. For MnZn ferrite cores, thermal conductivity ranges from $\sigma_c = 35$ to 42 mW/cm \cdot °K, with a nominal value of 25 mW/cm \cdot °K. Temperature rise is limited to $\Delta T = 40^\circ\text{C} = 40^\circ\text{K}$. The thermal resistance of air, according to the general formula for convective heat transfer, is:

$$R_{cA} = \frac{1}{h_{cA} \cdot A} = \frac{1}{h_{cA} \cdot (4 \cdot \pi \cdot r^2)}$$

where, for air, the convection coefficient is taken to be $h_{cA} \approx 2.5$ mW/cm 2 \cdot °K.

Combining these thermal resistances:

$$R_\theta = \frac{1}{4 \cdot \pi \cdot r} \cdot \left(\frac{1}{\sigma_c} + \frac{1}{h_{cA} \cdot r} \right) = \frac{1}{4 \cdot \pi \cdot r} \cdot \left(25 \frac{\text{cm} \cdot \text{K}}{\text{W}} + \frac{400 \frac{\text{cm}^2 \cdot \text{K}}{\text{W}}}{r} \right)$$

This expression can be solved for r from which spherical volume can be calculated:

$$r = \frac{1}{4\pi \cdot R_\theta} \cdot \left(\frac{1}{2 \cdot \sigma_c} + \sqrt{\left(\frac{1}{2 \cdot \sigma_c} \right)^2 + \frac{4\pi \cdot R_\theta}{h_{cA}}} \right)$$

We now have a relationship between P_c and V which can be expressed explicitly as:

$$V = \frac{4}{3} \cdot \pi \cdot \left[\frac{P_c}{4\pi \cdot \Delta T} \cdot \left(\frac{1}{2 \cdot \sigma_c} + \sqrt{\left(\frac{1}{2 \cdot \sigma_c} \right)^2 + \frac{4\pi \cdot \Delta T}{h_{cA} \cdot P_c}} \right) \right]^3$$

Core power density can also be calculated from r . By combining previous expressions:

$$p_c = \frac{\sigma_\theta \cdot \Delta T}{l} \cdot \frac{A}{V}$$

For the spherical approximation, then:

$$p_c = \frac{\sigma_c \cdot \Delta T}{r} \cdot \frac{A}{V} = \frac{\Delta T}{(8.33 \text{ cm} \cdot \text{K/W}) \cdot r^2 + (133 \text{ cm}^2 \cdot \text{K/W}) \cdot r}$$

where, $r = \left(\frac{3 \cdot V}{4 \cdot \pi} \right)^{1/3}$

What would be the allowable p_c for an EE25-25 core, which has a volume of 1.92 cm³? Calculating first, $r = 0.77$ cm, and then:

$$p_c(\text{EE24-25}) = 371 \text{ mW/cm}^3$$

Some other cores, taken from Nicera data, are:

Core Type	V , cm ³	r , cm	p_c , mW/cm ³
EER-28	5.83	1.12	252
E21	11.8	1.41	196
EC70	39.6	2.11	126

Core shapes are thermally suboptimal relative to a sphere. Consequently, these values of p_c should be taken as an optimistic upper bound.

The allowable p_c decreases with V because power is being dissipated everywhere within an increasing volume, and as r increases, surface A does not increase as quickly as V . Smaller dissipating cores *get the heat out* better than larger ones, and it is no surprise to find that an EC70 core is about as large as is available in commercial magnetics catalogs.

The assumed value of P_c used to calculate r (and V) is some fraction of total primary power. If the losses in the transductor are optimally divided between winding and core, then their power losses will be approximately equal (and exactly equal for a linear transductor). Typically, transductor efficiency, η , is around 98%, and core loss is therefore about 1% of the total conversion power. The 5 to 15 W/cm³ value given in commercial literature then results in $p_c = (15 \text{ W/cm}^3) \cdot (0.01) = 150 \text{ mW/cm}^3$. The range of 50 to 150 mW/cm³ is a familiar range for p_c . It corresponds to a mid-sized core, roughly the E21 in the above table (keeping in mind that the table values are an optimistic minimum). The volume-independent rule of thumb assumes a fixed η at a midrange V .

All that is required for iterative design is that a reasonable initial value of V be chosen, and for typical transductor η , the above formula for V appears to be. Upon calculation of winding and magnetic losses, total loss is then known and can be multiplied to R_θ , calculated from the above approximation, to determine whether ΔT is within bounds.

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