

Design Techniques For New Engineers

Transductors: Magnetic Devices

by Dennis L Feucht

Transformers, or coupled inductors, which are often referred to more generally as *magnetic devices* and which I prefer to call by the more economical neologism, *transductors*, can often be confusing to new and not-so-new engineers alike. Textbook coverage of them in school is sparse in the passive networks course, and can even tend to confuse the subject. Motor courses start with transformer magnetics and are better, but often do not highlight a critical concept, that of the different frames of reference.

Consider a two-winding transductor. What is its inductance? If primary and secondary inductances, L_p and L_s are measured and if the turns ratio:

$$n = \frac{N_p}{N_s}$$

is other than one, the inductance values will differ and be related by:

$$n = \sqrt{\frac{L_p}{L_s}}$$

Where did this come from? We must go to the beginning of the topic and derive basic transductor theory from first principles.

Magnetic Reference Frames

Transductor circuit quantities can be of the primary or secondary circuit. When the secondary circuit is *referred* to the primary (some engineers say *reflected*, but *referred* is the preferred terminology), the secondary circuit impedances are transformed using the turns ratio, n , which is the *referral ratio*, as impedances, by multiplying secondary impedances by n^2 . The resulting equivalent circuit offers a primary-side view of the secondary circuit.

The simplest case of a magnetic device is that of the inductor. Inductors have an electrical winding, usually with multiple turns, N . The various magnetic and electrical relations interplay in a way that suggests a *circuit* reference-frame and a magnetic *field* reference-frame. The terminal electrical quantities are in the circuit frame. For instance, current, i , flowing through the inductor at its terminals does not “appear” to the field as i but as $N \cdot i$. Each of the N loops of current adds to the total field flux an amount ϕ of flux. To the field, the voltage across each turn is v/N , but at the inductor terminals, the applied voltage is v , the sum of the per-turn voltages. The following table summarizes the two frames of reference.

Reference-Frame	Current	Inductive Quantity	Magnetic Flux	Voltage
electrical circuit (terminal quantities)	i	inductance, L	$\lambda = N \cdot \phi$	v
magnetic field	$N \cdot i$ (MMF)	per-turn-squared inductance, or permeance, L	ϕ	v/N

From the terminals (circuit frame), the flux referred to the circuit, or *circuit flux* is often called the *flux linkage*:

$$\lambda = N \cdot \phi$$

This circuit quantity relates the circuit current and inductance:

$$\lambda = L \cdot i$$

The corresponding equation, referred to the field, is the magnetic-Ohm's-Law (M Ω L):

$$\phi = L \cdot (N \cdot i)$$

The field flux, ϕ , is λ referred to the per-turn field reference-frame. The permeance, L , is the per-turn-squared or *field-referred inductance*, and $N \cdot i$ is the *field-referred current*, or MMF.

The above two equations are equivalent but expressed in circuit and field reference-frames. The relating quantity is the number of turns, N . By multiplying each side of M Ω L by N and substituting:

$$N \cdot \phi = N \cdot L \cdot (N \cdot i)$$

or,

$$\lambda = N^2 \cdot L \cdot i = L \cdot i$$

and,

$$L = N^2 \cdot L$$

The permeance symbol, L , in magnetics manufacturers' core data is typically the non-mnemonic symbol, AL . It is the field-referred inductance. The circuit inductance is related to the field inductance by N^2 .

The field current, $N \cdot i$, and per-turn-voltage of v/N transform the circuit impedance to a field impedance of:

$$Z_{field} = \frac{v/N}{N \cdot i} = \frac{Z_{circuit}}{N^2}$$

Each turn has an applied or induced voltage of v/N while enclosing a field through a loop with an equivalent current of $N \cdot i$. Current viewed in the field frame, or field-referred current, is multiplied by N while the field-referred voltage is divided by N relative to the inductor circuit terminals.

The familiar n^2 turns-ratio transform of transformer impedances corresponds to N^2 for single-winding inductors. This circuit-field transform of reference-frames is more basic and general than for transformers alone, and also applies to circuit- and field-referred quantities in inductors.

Transductor Mutual Inductance

Transductors have two or more windings (two in this explanation) which are magnetically coupled; they share some magnetic field. The inductance of each winding can be decomposed into that which does not share flux with the other winding, called *leakage inductance*, and the remainder which does, called magnetizing inductance. The total winding inductance is the self-inductance:

$$L = L_{leakage} + L_{magnetizing} = L_l + L_m$$

Mutual inductance is the transductor interwinding inductance:

$$M = \frac{\lambda_{12}}{i_2} = \frac{\lambda_{21}}{i_1}$$

where, windings 1 and 2 have circuit fluxes, λ .

The circuit-referred flux of winding 1 as produced by the current in winding 2, or i_2 , is λ_{12} , as given in the first right-hand expression above. The symmetry of linear magnetics also results in the two expressions being equal.

The circuit fluxes in the above expressions can be referred to the transductor field by the referral relationship, $\lambda = N \cdot \phi$, where N is the number of turns:

$$M = N_1 \cdot \frac{\phi_{12}}{i_2} = N_2 \cdot \frac{\phi_{21}}{i_1}$$

Then substituting for ϕ using MQL:

$$M = N_1 \cdot \frac{\mathbf{L}_m \cdot (N_2 \cdot i_2)}{i_2} = N_2 \cdot \frac{\mathbf{L}_m \cdot (N_1 \cdot i_1)}{i_1}$$

This quickly reduces to:

$$M = N_1 \cdot N_2 \cdot L_m$$

The mutual inductance is the field-referred inductance (permeance) times each of the winding turns. It is best thought of as an inductance shared by the two windings – an interwinding inductance – for it is not associated with the circuit inductances of either winding but includes the turns of both. It is an inductance internal to the transductor, in an interwinding frame of reference.

The winding (self-) inductances are derived as follows. By the definition of inductance, it is circuit-referred:

$$L = \frac{\lambda}{i} = \frac{N \cdot \phi}{i} = \frac{N \cdot (\mathbf{L}_m \cdot N \cdot i)}{i} = N^2 \cdot \mathbf{L}_m$$

Then the mutual inductance, referred to the windings 1 and 2 terminals, can be expressed in M as magnetizing inductances;

$$L_{m1} = N_1^2 \cdot \mathbf{L}_m = \left(\frac{N_1}{N_2} \right) \cdot M ; L_{m2} = N_2^2 \cdot \mathbf{L}_m = \left(\frac{N_2}{N_1} \right) \cdot M$$

Expressed using the turns ratio, these are:

$$L_{m1} = n \cdot M$$

$$L_{m2} = \frac{M}{n}$$

$$L_{m1} = n^2 \cdot L_{m2}$$

The last equation relates the 1 and 2 winding inductances, as it appeared earlier, solved for n:

$$n = \sqrt{\frac{L_{m1}}{L_{m2}}}$$

Expressing M in winding magnetizing inductances:

$$M = \frac{L_{m1}}{n} = n \cdot L_{m2} = \sqrt{L_{m1} \cdot L_{m2}}$$

The coupling between windings is not entirely due to leakage fluxes of each winding. They can be included by a transductor parameter, the coupling coefficient, k, defined by the following equations:

$$\left. \begin{aligned} L_{m1} &= k \cdot L_1 \\ L_{m2} &= k \cdot L_2 \end{aligned} \right\}, k \geq 0$$

The coupling coefficient is defined as always positive in value. Substituting:

$$M = \sqrt{L_{m1} \cdot L_{m2}} = k \cdot \sqrt{L_1 \cdot L_2}$$

Furthermore, the leakage inductances of the windings are:

$$L_{l1} = L_1 - L_{m1} = L_1 - k \cdot L_1 = (1 - k) \cdot L_1$$

$$L_{l2} = L_2 - L_{m2} = L_2 - k \cdot L_2 = (1 - k) \cdot L_2$$

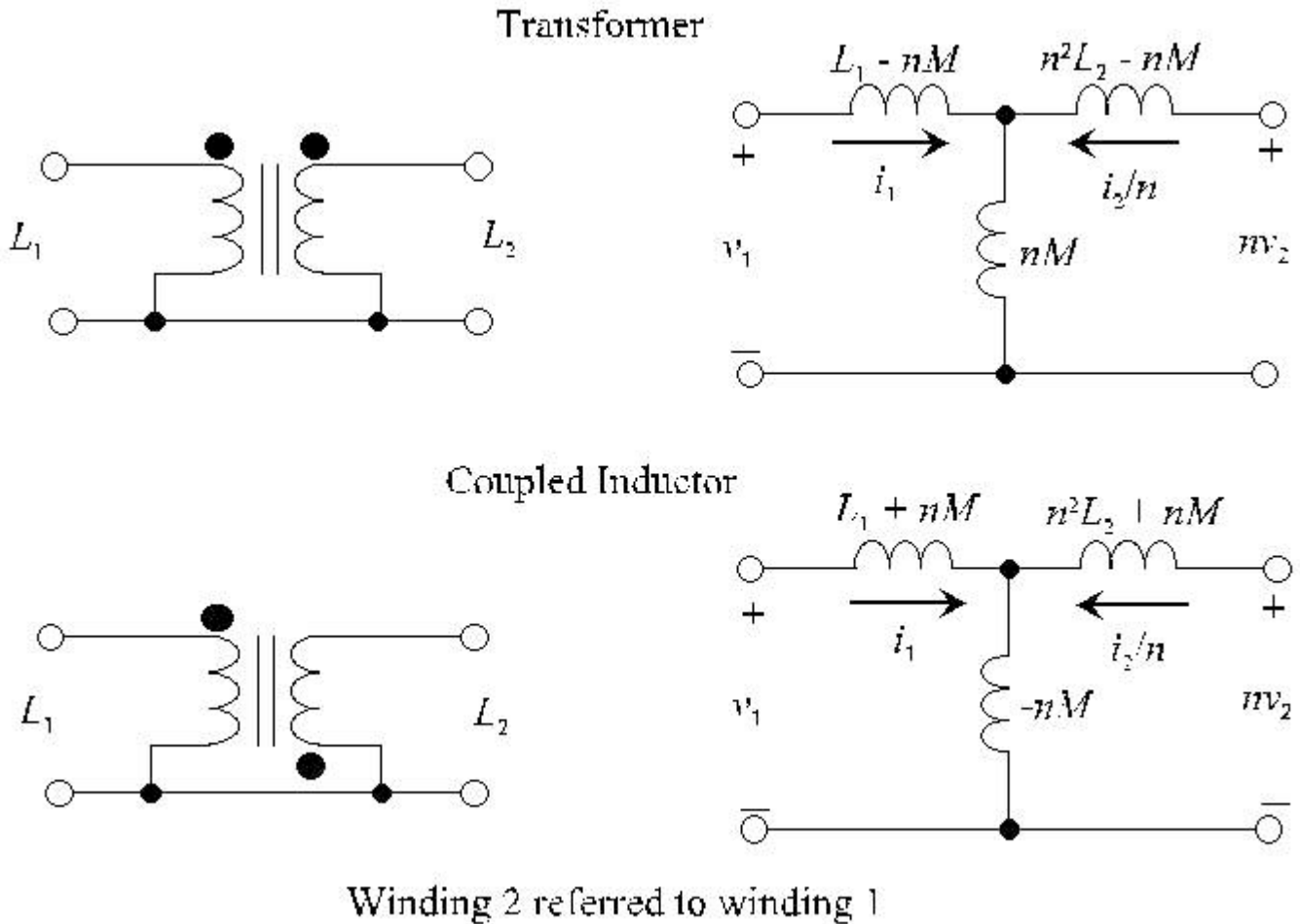
Additionally:

$$n = \sqrt{\frac{L_{m1}}{L_{m2}}} = \sqrt{\frac{L_{m1}/k}{L_{m2}/k}} = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{L_{m1}/(1-k)}{L_{m2}/(1-k)}} = \sqrt{\frac{L_{l1}}{L_{l2}}}$$

The value of $M \geq 0$, though the mutual or magnetizing inductance in a transductor circuit model is bipolar. When the winding fluxes add (or aid), the transductor is a coupled inductor and $-n \cdot M < 0$. When the fluxes oppose, it is a transformer for which $+n \cdot M > 0$. By adding the correct sign to M , M remains positive. Then in either case:

$$k = \frac{M}{\sqrt{L_1 \cdot L_2}}$$

Three-terminal device models for transductors are shown below for both polarities of M , and with winding 2 referred to winding 1.



Transducer Circuit Models

The basic transducer circuit models shown above do not preserve the isolation of the windings but are constrained by a common terminal. Besides this, the above models are only one possibility depending on referral ratio, a . Ordinarily, $a = n$, but this does not have to be so. Three possibilities, beginning with the most familiar, are given below for a transformer with winding 2 referred to winding 1 by referral ratio a . A winding 2 quantity referred to winding 1 is indicated by a prime, as in L_2' . In general, the winding 2 impedance referred to the winding one side is:

$$Z_2' = \frac{a \cdot v_2}{i_2 / a} = a^2 \cdot Z_2$$

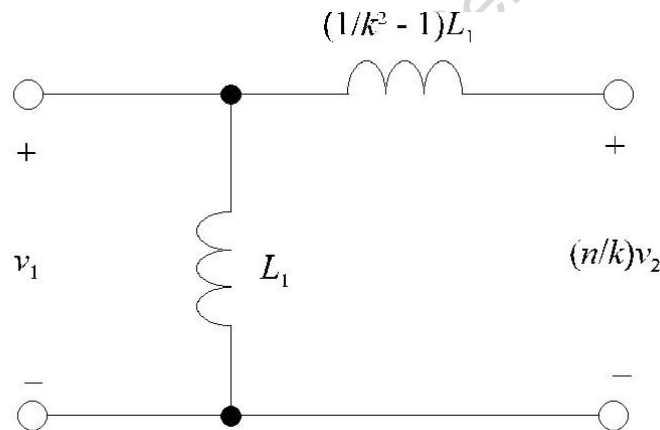
For the first case, split the leakage inductance evenly between windings. Then:

$$L_{11} = L_{22}' \Rightarrow L_1 - a \cdot M = a^2 \cdot L_2 - a \cdot M \Rightarrow a = \sqrt{\frac{L_1}{L_2}} \Rightarrow L_{11} = (1 - k) \cdot L_1 ; L_{22}' = (1 - k) \cdot L_1 ; a \cdot M = k \cdot L_1 ; a = n$$

For the second case, refer all the leakage inductance to winding 2 by setting:

$$L_1 - a \cdot M = 0 \Rightarrow a = \frac{L_1}{M} = \frac{1}{k} \cdot \sqrt{\frac{L_1}{L_2}} \Rightarrow L_{11} = 0 ; L_{22}' = (1/k^2 - 1) \cdot L_1 ; a \cdot M = L_1 ; a = n/k$$

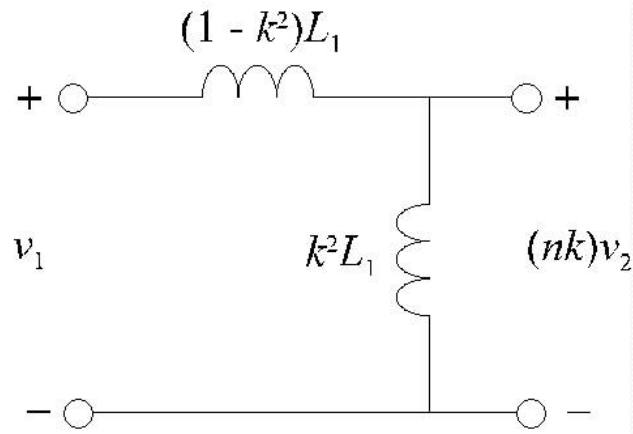
The resulting model is shown below. All leakage inductance is on the winding 2 side.



The third case refers all the leakage inductance to the primary winding by setting:

$$a^2 \cdot L_2 - a \cdot M = 0 \Rightarrow a = \frac{M}{L_2} = k \cdot \sqrt{\frac{L_1}{L_2}} \Rightarrow L_{11} = (1 - k^2) \cdot L_1 ; L_{22}' = 0 ; a \cdot M = k^2 \cdot L_1 ; a = n \cdot k$$

The equivalent circuit is shown overleaf.



Because leakage inductances behave as though they were external series inductances, they can be referred to the other winding in the same way as any external impedance in the circuit. Analyses can be simplified by choosing the optimal model. The above transformer variations can also be worked out for when winding 1 is referred to winding 2; the equations are the same, with 1 and 2 subscripts interchanged.



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