What Is A *g* Unit?

by Wayne Tustin

Equipment Reliability Institute, Santa Barbara, California

Has someone asked you to measure vibratory or mechanical shock acceleration? He or she probably wants you to report acceleration (or deceleration) intensity in *g* multiples. Has someone asked you to perform a sinusoidal vibration test? He or she probably states the desired test vibration intensity in *g* multiples. Has someone asked you to perform a random vibration test? He or she probably states the desired test vibration intensity in rather strange g^2/Hz units.

Hopefully, you and that person mean precisely the same thing when you place the italic letter *g* after a number. What is that meaning?

Instantaneous and Peak Displacement

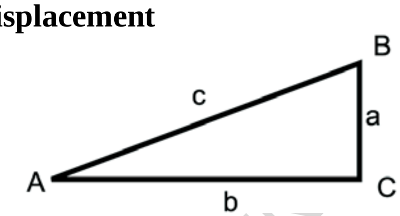

Fig. 1: Trigonometry Review

Examining Fig. 1, visualize line c rotating. It started at 0º and has rotated about 30º to where we see it in Figure 1. It will continue to rotate to a point directly over point A (90º) and continuing around a 360º circle not just once but many times. Radius c = AB is constant, but length a will change from zero to a positive maximum (equal to c) then fall to zero and commence rising again (but in a negative or downward direction) to a negative maximum (again equal to c) and then fall back to zero. This sequence repeats, each rotation.

Let length a represent an alternating voltage which represents a mechanical vertical oscillation of point B. Let instantaneous voltage x represent B's instantaneous height relative to point C. x is the instantaneous length of a (see x in Fig. 2).

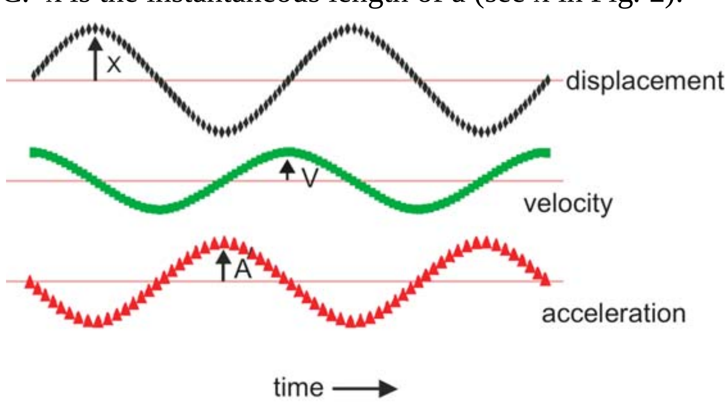

Fig. 2: Peak Values of Displacement, Velocity, and Acceleration

We can write an equation:

$$x = X\sin 2\pi ft$$

to describe the variations in voltage x. X is the maximum value (peak value) of that voltage.

Velocity

Let us now differentiate or take the slope or rate of change of x, and call the result our instantaneous velocity v. See the second graph of Fig. 2, with the instantaneous value:

$$v = dx/dt = 2\pi fX\cos 2\pi ft$$

Better, since most metrologists use peak-to-peak displacement D rather than zero-to-peak X:

$$v = dx/dt = \pi fD\cos 2\pi ft$$

We seldom use v. But we do need V, its maximum value. We select the instant when v = V, so that V = πfD.

Imperial/USA Unit Velocity Example:

Let f = 100 Hz and D = 0.1 inch. Then V = (π) (100) (0.1) = 31.4 inch/s.

Metric System Unit Velocity Example:

Let f = 100 Hz and D = 2.54 mm. Then V = (π) (100) (2.54) = 798 mm/s or 0.798 m/s.

Acceleration

To get an equation for a, the instantaneous acceleration, (lowest trace of Fig. 2) we differentiate or "get the slope of" our earlier equation:

$$v = dx/dt = \pi fD\cos 2\pi ft$$
$$a = dv/dt = 2\pi^2 f^2 D\sin 2\pi ft$$

We seldom use a. But we do need A, its maximum value. We select the instant when a = A, so that $A = 2\pi^2 f^2 D$.

Imperial/USA Unit Acceleration Example:

Let f = 100 Hz and D = 0.1 inch. Then A = (2) (π) (π) (100) (100) (0.1) = 19,740 inch/s^2. We compare that with the widely recognized acceleration standard (the Earth's gravitational acceleration at sea level, *g*), namely 32.2 feet/s^2 or 386 inch/s^2 , thus A = 19,739 ÷ 386 = 51.1*g*. We can simplify the foregoing by using:

$$A = 2\pi^2 f^2 D \div 386 = 0.0511 f^2 D$$

Metric System Unit Acceleration Example:

Let f = 100 Hz and D = 2.54 mm.

Then A = (2) (π) (π) (100) (100) (2.54) = 501,376 mm/s^2 or 501.4 m/s^2. We compare that with the widely recognized acceleration standard (the Earth's gravitational acceleration at sea level, *g*), namely 9807 mm/s^2, thus, A = 50,1376 ÷ 9807 = 51.1*g*. We can simplify the foregoing by using:

$$A = 2\pi^2 f^2 D \div 9807 = 0.00202 f^2 D$$

Acceleration standards

Where did those standard values, 32.2 feet/s^2 or 386 inch/s^2 and 9.807 m/s^2 or 9807 mm/s^2, come from?   From careful observation of instantaneous velocity when objects are dropped. They accelerate. Their velocity increases by 32.2 feet/s or 386 inch/s or 9.807 m/s or 9807 mm/s, each second of free fall.

Those numbers represent 1*g*.

In addition to the foregoing relations between f, D and A, you might need relations between f, V and A. Divide the A, f, D equation by the V, f, D equation, thus, with A in *g*, V in inch/s and f in Hz:

$$A \div V = 0.0511 f^2 D \div \pi f D = 0.0162 f,$$
$$\text{so } A = 0.0162 fV$$

In Metric, A in *g*, V in m/s and f in Hz:

$$A \div V = 0.00202 f^2 D \div \pi f D = 0.00064 f,$$
$$\text{so } A = 0.00064 fV$$

Or, with A in m/s^2, V in m/s and f in Hz:

$$A/V = 2\pi^2 f^2 D / \pi f D = 2\pi f$$

The foregoing mathematics is eased by cardboard vibration calculators such as the one shown (overleaf) in Fig. 3. These are commonly given away by shaker and accelerometer manufacturers. Most will fit into a jacket pocket. They save memorizing the foregoing equations, but I recommend that you do memorize them and use an electronic calculator, as they are much more accurate than the cardboard calculators.

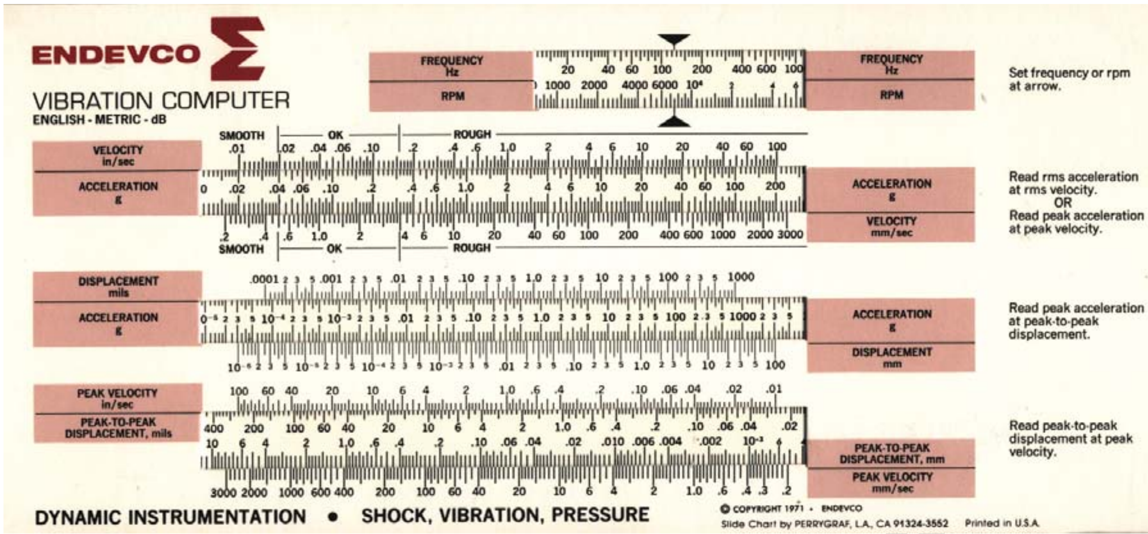

Fig. 3: Cardboard Vibration Calculator

Visit the Web sites of shaker and accelerometer manufacturers. Several offer on-line calculators (some can be downloaded) and CDs yielding a display such as Fig. 4 and permitting these and other useful calculations.

What is 1*g*? Not only is it 32.2 feet/s^2 or 386 inch/s^2, but it's also the velocity increase, 32.2 feet/s or 9.8 m/s you'd experience in each second of free fall, if you stepped off a building. Amusement park rides accelerate/decelerate you less than 4*g*, to avoid injury.

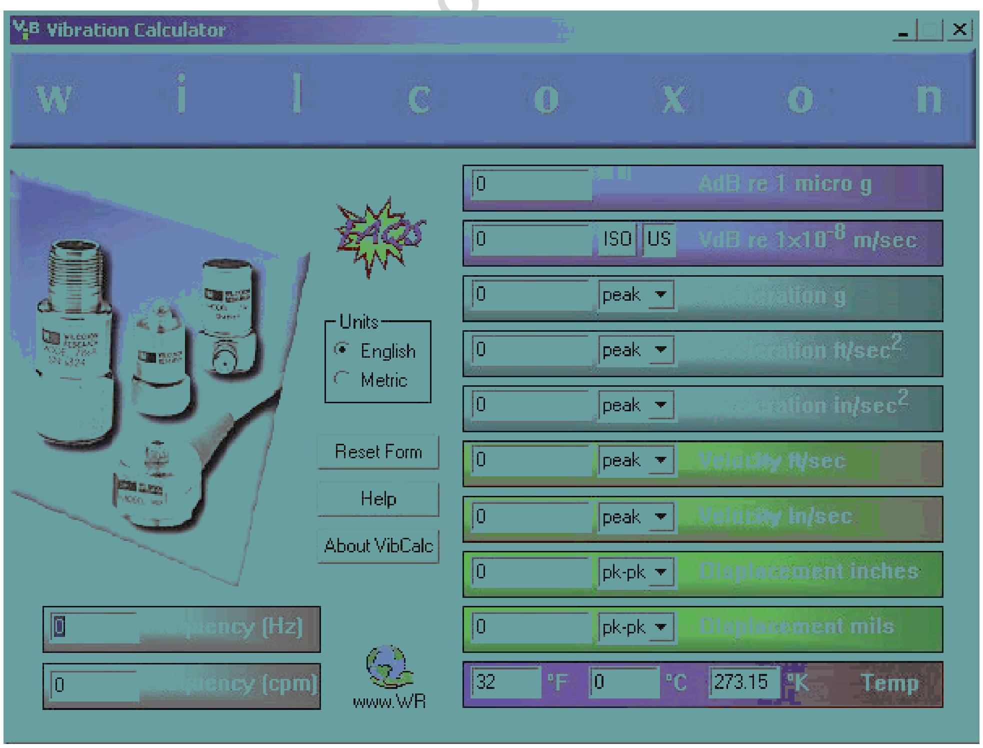

Fig. 4: CD-ROM Vibration Calculator